

Häirekindlad koodid

$$2^n \rightarrow 2^{n+d}$$

2^n - lubatud kombinatsioonid

$2^{n+d} - 2^n$ - keelatud kombinatsioonid

d - liiasus

$$\frac{d}{n} \quad \frac{d}{n+d}$$

Vigade avastamine

Vigade parandamine

Koodikaugus (Hamming)

$$\Delta(A, B) \Rightarrow \#(A \oplus B)$$

$$\Delta_{\min} = \min \Delta(A, B)$$

Vigade avastamine:

$$d_{\min} \geq t+1$$

t - vea kordsus

Vigade parandamine

$$d_{\min} = 2t + 1$$

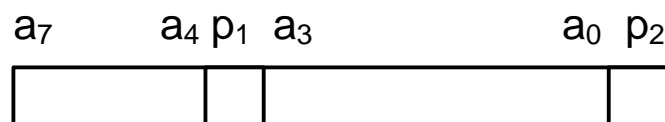
N: Paarsuskontroll

$$a_8 = \bigoplus_{i=0}^7 a_i \quad \text{või} \quad (\text{paarsus})$$

$$a_8 = \neg \left(\bigoplus_{i=0}^7 a_i \right) \quad (\text{paaritus})$$

Ei avasta 0-vektorit (paarsus)

1-vektorit (paaritus)

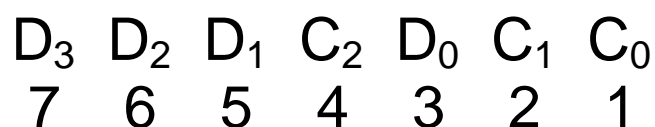


$$p_1 = \bigoplus_{i=0}^7 a_i \qquad p_2 = \neg \left(\bigoplus_{i=0}^3 a_i \right)$$

N: Hammingu koodid

$$2^n \rightarrow 2^{n+p} \quad (\text{"check" bitid})$$

$$2^4 \rightarrow 2^{4+3}$$



"Check" bitid $\rightarrow 2^k$, $k = 0, 1, 2, \dots$

Nr	Bit	C_2	C_1	C_0
3	D_0	0	1	1
5	D_1	1	0	1
6	D_2	1	1	0
7	D_3	1	1	1

$$C_0 = D_0 \oplus D_1 \oplus D_3$$

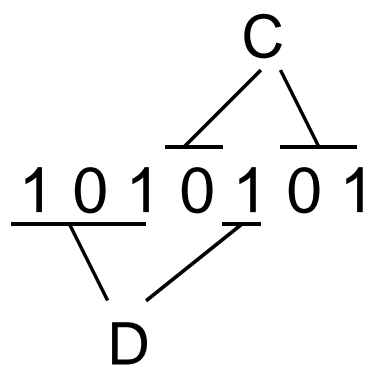
$$C_1 = D_0 \oplus D_2 \oplus D_3$$

$$C_2 = D_1 \oplus D_2 \oplus D_3$$

$$D = 1\ 0\ 1\ 1 \Rightarrow C_0 = 1$$

$$C_1 = 0$$

$$C_2 = 0$$

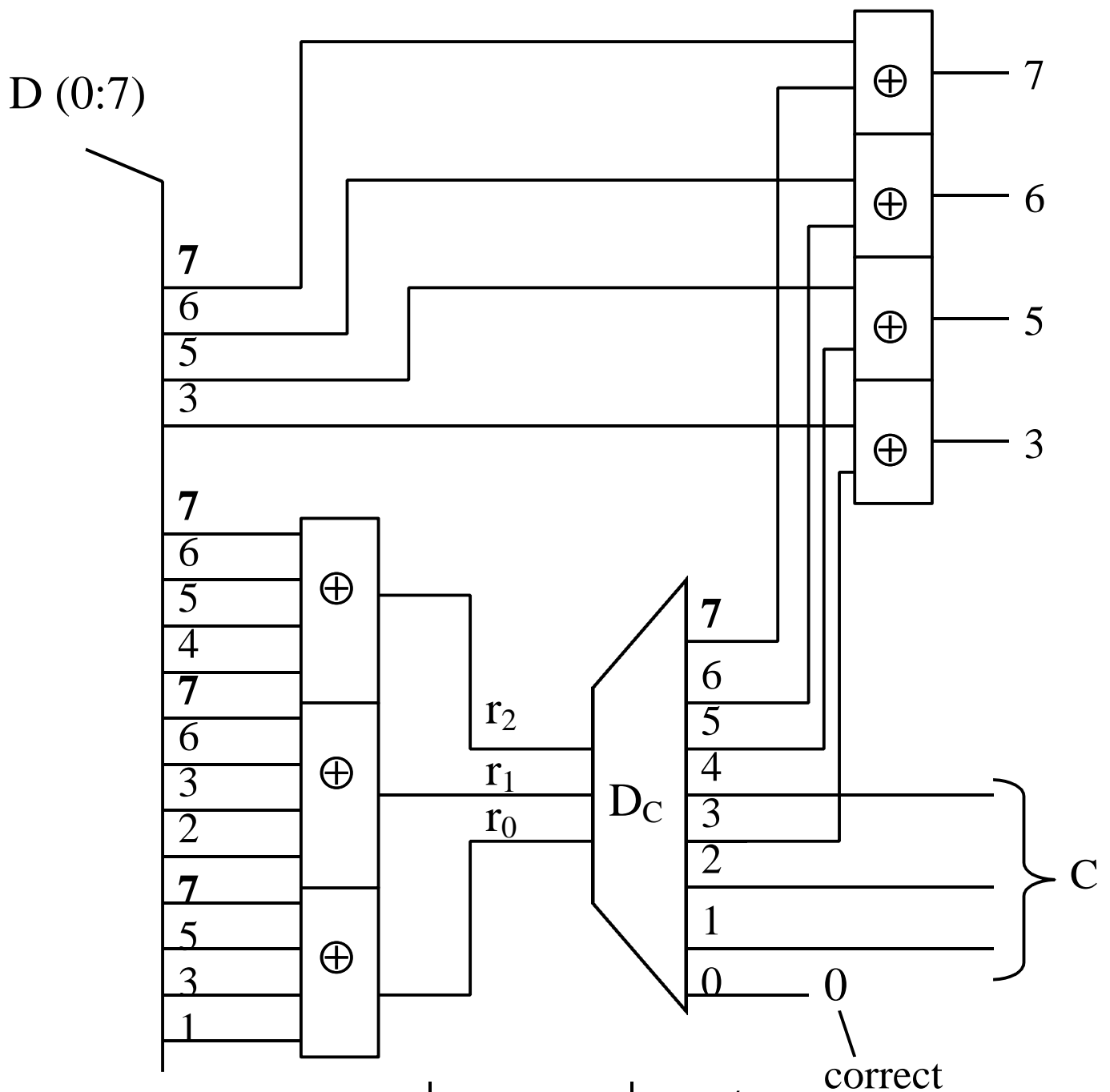


$$\text{Syndrom : } r_0 = C_0 \oplus D_0 \oplus D_1 \oplus D_3$$

$$r_1 = C_1 \oplus D_0 \oplus D_2 \oplus D_3$$

$$r_2 = C_2 \oplus D_1 \oplus D_2 \oplus D_3$$

Syndrom	Result
0 0 0	Correct
1 0 0	Error in C
0 1 0	
0 0 1	
0 1 1	error D ₀
1 0 1	error D ₁
1 1 0	error D ₂
1 1 1	error D ₃



n	p	p/n
4	3	75%
8	4	50%
16	5	~30%
32	6	~20%
64	7	~10%

64 bitti C \rightarrow 1, 2, 4, 8, 16, 32, 64

Puudus : ei erista 1x ja 2x vigu

Lisame paarsusbiti PB

PB	Syndrom	Result
Corr.	Corr.	OK
Error	Corr.	Single error in PB
Error	Error	Single error in PB, parandata
Corr.	error	Double error

N : Data 8 bitti

Paarsus 1 bitt (13)

Liiasus 4 bitti (1, 2, 4, 8)

13	12	11	10	9	8	7	6	5	4	3	2	1
PB	D ₇	D ₆	D ₅	D ₄	C₃	D ₃	D ₂	D ₁	C₂	D ₀	C₁	C₀

$$C_3 = D_4 \oplus D_5 \oplus D_6 \oplus D_7$$

$$C_2 = D_1 \oplus D_2 \oplus D_3 \oplus D_7$$

$$C_1 = D_0 \oplus D_2 \oplus D_3 \oplus D_5 \oplus D_6$$

$$C_0 = D_0 \oplus D_1 \oplus D_3 \oplus D_4 \oplus D_6$$

i	Bitt	C ₃	C ₂	C ₁	C ₀
3	D ₀	0	0	1	1
5	D ₁	0	1	0	1
6	D ₂	0	1	1	0
7	D ₃	0	1	1	1
9	D ₄	1	0	0	1
10	D ₅	1	0	1	0
11	D ₆	1	0	1	1
12	D ₇	1	1	0	0

Infobittide arvu võib tõsta;

koodid 1 1 0 1
 1 1 1 0
 1 1 1 1 kasutamata.

Moodulkontroll

$$A \Rightarrow a_{n-1} a_{n-2} \dots a_2 a_1 a_0$$

A jagatakse n-bitisteks gruppideks

$$A \Rightarrow \alpha_k \alpha_{k-1} \dots \alpha_1 \alpha_0$$

$$\alpha_i \Rightarrow 0 \dots (2^m - 1)$$

$$A \pmod{(2^m - 1)} = \sum_i \alpha_i \pmod{2^m - 1}$$

$$N : \quad 47 \pmod{7} = 5$$

$$47_{10} = 57_8$$

$$(5+7) \pmod{7} = 5$$

Kontrollgrupp:

$$A \pmod{(2^m - 1)} + K_A = 2^m - 1$$

$$(A + K_A) \pmod{(2^m - 1)} = 0$$

$$57_8 \Rightarrow \underbrace{1\ 0\ 1}_{K_A} \underbrace{1\ 1\ 1} \underbrace{0\ 1\ 0}$$

$$572 = (5 + 7 + 2) \pmod{7} = 0$$

$$572_8 = 378_{10} \Rightarrow 378 \pmod{7} = 0$$

Modul 3 kontroll:

$$(A + K_A) \pmod{3} = 0$$

$$N : \quad A = 1\ 1\ 0\ 1\ 1\ 0\ 1\ 0\ 1\ 1$$

$$K_A = 0\ 1$$

Summeerimine:

$$C = A + B$$

$$A + K_A + B + K_B \Rightarrow C + K_C$$

N :

A: 00 11 01 10 10 11 01 K_A

B: 00 00 10 11 01 01 10 K_B

C: 01 00 00 10 00 00 11 K_C

NB! $K = 3$ $K = 0$